Diffractions of Bose-Einstein Condensate in Quantized Light Fields

Peng Zhang,[1] Z.-Y. Ma,[2] Jian-Hua Wu,[1] H. Fan,[1] and W. M. Liu[1]

[1]*Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*

[2]*Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China*

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We investigate the atomic diffractions of a Bose-Einstein condensate in quantized light fields. Situations in which the light fields are in number states or coherent states are studied theoretically. Analytical derivation and numerical calculation are carried out to simulate the dynamics of the atomic motion. In condition that atoms are scattered by light in the number states with imbalanced photon number distribution, the atomic transitions between different momentum modes would sensitively depend on the transition order and the photon number distribution. The number-state-nature of the light fields modifies the period of atomic momentum oscillations and makes forward and backward atomic transitions unequal. For light fields in coherent states, no matter the intensities of the light fields are balanced or not, the atomic diffractions are symmetric and independent on the transition order.

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I. INTRODUCTION

The coherent interaction between matter and electromagnetic fields within various kinds of physical system is providing a useful platform for developing concepts in quantum optics and atomic molecular physics. The experimental realization of Bose-Einstein condensation in dilute atomic gases [1], greatly facilitates the investigations of the interactions between the atom and light on a macroscopic scale [2–6]. In the past few years, atomic scattering process in optical fields has been intensively studied, such as matter-wave supperradiance [7–9], quantum phase transition [10, 11], cavity optomechanics [12, 13], raising a variety of striking discoveries.

In physical systems involving light-atom interactions, various kinds of experimental configurations have been developed or proposed to achieve the desired atom-field coupling [21, 22]. Especially, by using high-quality resonators [14–16], strong coupling regime of the light-atom can be reached, where atoms coherently exchange photons with light fields. However, in the previous light-atom interacting models, the derivations usually use the classical treatment for the electromagnetic fields, in which the light fields are recognized as amplitude-modulated plane waves with slowly varying amplitudes; or even in a quantized treatment for light fields, mean-field approximations is often introduced for the light fields. Atomic diffractions in such situations have been well studied, however, in most conditions, the subtle effects of the atomic motion induced by the quantum nature of light has always been ignored.

In this paper, we study the diffractions of a Bose-Einstein condensate (BEC) in multimode optical fields, with a full quantum treatment of the light fields. In the cases that the quantized light fields are in different kinds of states, exotic behaviors are induced in the atomic motion by the quantum-nature of light. This paper is organized as following, in section II, we introduce the physical system and develop the theoretical model of the atomic diffractions in a full quantum treatment of the light fields. In section III, we analyze how the quantum status of the pump fields can modify the atomic motion in the cases that the optical fields are in number states. In section IV, we turn to the case where the light fields are in coherent states. In section V, we arrive at a summary of our results.

II. THE THEORETICAL MODEL FOR ATOMIC DIFFRACTION IN QUANTIZED LIGHT FIELDS

The physical system under study is illustrated in Fig. 1, where an elongated BEC is trapped along the horizontal axis of a triangle ring cavity. The cavity consists of three mirrors which are placed in a way such that the light reflected inside the cavity forms a closed loop. There are two mutually counterpropagating modes, i.e., the clockwise and anti-clockwise modes, coupled equally to the atoms. The light fields in these two modes are also referred as the forward (or right-going, with wave-vectork_R) and backward (or left-going, with wave-vector k_L) ones according to the geometrical configuration. These two cavity modes are degenerated in frequency, $\omega_R = \omega_L = \omega_c$, so the wave-vectors of the two cavity mode satisfies $k_R = -k_L$. The atoms within the condensate are recognized as two-level atoms with one internal ground state $|g_{in}\rangle$ and excited state $|e_{in}\rangle$ representing the internal degrees of freedom for each atom. In the large detuning limit, the upper internal state $|e_{in}\rangle$ of the atoms can be adiabatically eliminated [9, 17, 18]. For illustration, the condensate is chosen to be prepared with ^{87}Rb atoms, and the wave length of the light fields is $\lambda = 780$nm with a detuning $\Delta = \omega_c - \omega_a = -1.5$GHz of the D_2 transition [8, 21, 22]. The Hamiltonian of the

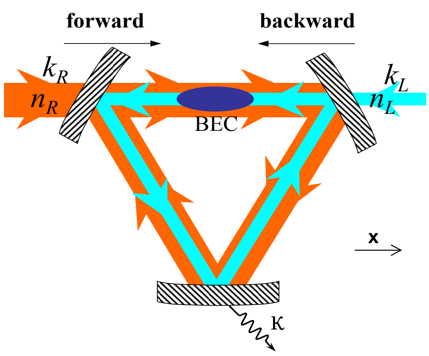

FIG. 1: (Color online) A schematic diagram of a two-mode triangle ring cavity containing an elongated Bose-Einstein condensate trapped along one of its cavity axis. The damping rate κ of the cavity modes is small compared to the atomic motion inside the cavity. There are two degenerate light modes propagating in the clockwise and anticlockwise directions inside the cavity.

light-atom coupling system can be written as,

$$\hat{\mathscr{H}} = \int d^3r \hat{\Psi}^\dagger(\mathbf{r})\Big[-\frac{\hbar^2}{2M}\nabla^2 + \hat{\mathbf{E}}(\mathbf{r})\cdot\hat{\mathbf{D}}(\mathbf{r})\Big]\hat{\Psi}(\mathbf{r}) + \frac{1}{2}\int d^3r d^3r' \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r}')V_{inter}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}')\hat{\Psi}(\mathbf{r}), \quad (1)$$

where $\hat{\Psi}^\dagger(\mathbf{r})$, $\hat{\Psi}(\mathbf{r})$ are the creation and annihilation field operators for the atomic field, respectively. $\hat{\mathbf{E}}(\mathbf{r})$ is the electric field operator of the electromagnetic field, and $\hat{\mathbf{D}}(\mathbf{r})$ is the atomic dipole moment operator. $V_{inter}(\mathbf{r}-\mathbf{r}')$ represents the interatomic potential. The counter-propagating light forms a standing wave in the triangle ring cavity. The atoms in the standing light feel a periodical potential. The condensate is transversely tightly bounded, therefore its transverse freedom is frozen, and it is reasonable to recognize the condensate as quasi-one-dimensional ensemble in its longitudinal direction [19]. This suggests that the matter field operators can be expanded with a set of Bloch functions, which are amplitude-modulated plane waves,

$$\hat{\Psi}(\mathbf{x}) = \sum_n \hat{b}_n e^{i\mathbf{k}_n\mathbf{x}} u(\mathbf{x}),$$
$$\hat{\Psi}^\dagger(\mathbf{x}) = \sum_n \hat{b}_n^\dagger e^{-i\mathbf{k}_n\mathbf{x}} u^*(\mathbf{x}), \quad (2)$$

where $u(\mathbf{x})$ is a periodical function along the direction the horizontal axis of the cavity, $u(x+\frac{\lambda}{2}) = u(x)$, with the period one-half of the wave-length of the light wave. $\hat{b}_n^\dagger$ and $\hat{b_n}$ are the creation and annihilation operators of the bosonic atoms in the nth Bloch mode. $\mathbf{k}_n = 2n\mathbf{k}_R$ is the wave vector of the modulated plane wave. For the absorption and subsequently stimulated emission of photons from the two modes of optical fields, the atoms would acquire a net momentum of even number multiples of photon momentum, $2n\hbar k_R$. Assuming the atoms are initially in the stationary state, after the scattering process, they will be excited to the higher adjacent momentum modes, which can be described as $(e^{2i\mathbf{k}_R\mathbf{x}} + e^{-2i\mathbf{k}_R\mathbf{x}})u_0(\mathbf{x})$, with $u_0(\mathbf{x})$ the envelop of the stationary wave packet. If the atoms get further scattered, some of them may go to the even higher momentum mode. For simplicity, we can approximate the amplitude-modulated plane waves with the wave functions of the free particles, and the wave functions of the atomic field now read,

$$|\Psi(x)\rangle = \sum_n \Psi_n \left|\frac{1}{\sqrt{V}}exp(\frac{i}{\hbar}p_n\cdot x)\right\rangle = \sum_n \Psi_n|p_n\rangle,$$
$$\langle\Psi(x)| = \sum_n \Psi_n^* \left\langle\frac{1}{\sqrt{V}}exp(\frac{-i}{\hbar}p_n\cdot x)\right| = \sum_n \Psi_n^*\langle p_n|. \quad (3)$$

In our case, due to the diluteness of the atomic gas, we are going to neglect the modification of wave functions from the atomic interation. Here, V is the volume of the condensate, $\langle x|p_n\rangle = \frac{1}{\sqrt{V}}\exp(\frac{i}{\hbar}p_n\cdot x)$ is the wave function of free particles with momentum $p_n = 2n\hbar k_R$ in coordinate space. The particle number of the atoms is normalized to one. At the same time, we have removed the hat from each operator of the matter field, using a mean-field treatment for the atomic field.

We approximate that the discrete modes of the atomic field makes a complete set describing dynamics of the condensate. Plugging Eq. (3) into Eq. (1), we arrive at the following expression for the Hamiltonian of the light-atom coupling system,

$$\hat{H} = \sum_{m=-\infty}^{\infty} \hbar\omega_m |p_m\rangle\langle p_m| + \hbar G \cdot \sum_{n=-\infty}^{\infty}\Big(a_{k_L}^\dagger a_{k_R}|p_n\rangle\langle p_{n-1}| + a_{k_R}^\dagger a_{k_L}|p_n\rangle\langle p_{n+1}|\Big), \quad (4)$$

where we have adopted the full quantized version for the electric component of the light fields, $\mathbf{E}(\mathbf{r}) = \sum_{\mathbf{k}} \hat{\epsilon}_{\mathbf{k}}\mathscr{E}_{\mathbf{k}}(a_{\mathbf{k}} + a_{\mathbf{k}}^\dagger), \mathbf{k} = k_{L,R}$, in which $\hat{\epsilon}_{\mathbf{k}}$ is the unit polarization vector; $\mathscr{E}_{\mathbf{k}} = \sqrt{\hbar\omega_c/2\epsilon V_c}$ has the dimension of an electric field with V_c the quantization volume of the macro-cavity [20]. $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ are the creation and annihilation operators of photons in the corresponding light mode, respectively. $\hbar\omega_m = 2(\hbar m k_L)^2/M$ is the energy of the mth unperturbed discretized mode, and M is the mass of a single atom within the condensate. $G = -\Omega^2/\triangle$ is the effective two-photon atom-field coupling constant, with $\Omega = -\frac{\mathbf{d}\cdot\hat{\epsilon}_{\mathbf{k}}\mathscr{E}_{\mathbf{k}}}{\hbar}$ the single-photon atom-field coupling constant and $\mathbf{d}$ the electric dipole moment of the atom. The form of G comes from the adiabatic elimination of the internal excited state of the two-level atom, and this is legitimated by large detuning of the light fields. This Hamiltonian is rather reminiscent of an ordinary lattice model in solid state physics in spite of an atomic interacting term. The second hopping term disturbs the motion of the free atoms and in-

duces exchanging of particles between different diffraction orders. Such atomic diffractions are categorized into Raman-Nath diffractions with short pulse fields and Bragg diffractions with longer time of light-atom interaction.

To investigate the atomic motion of the system, it is usually convenient to work in the interaction picture by seperating the Hamiltonian (4) into the free part and the atom-field interacting part, then one has,

$$\begin{aligned} \hat{\mathscr{V}}(t) &= \hbar G \sum_{n=-\infty}^{\infty} \{ a_{k_L}^{\dagger} a_{k_R} \left| p_n \right\rangle \left\langle p_{n-1} \right| e^{-i\delta_-(t)} \\ &+ a_{k_R}^{\dagger} a_{k_L} \left| p_n \right\rangle \left\langle p_{n+1} \right| e^{-i\delta_+(t)} \}, \end{aligned} \quad (5)$$

where $\delta_{\pm} = \pm 16\pi\nu_r(n \pm \frac{1}{2})$, and $\nu_r = \hbar k_L^2/(4\pi M)$ is the atomic one-photon recoil frequency ($\nu_r \sim 3.77$ kHz according to the parameters chosen in this system).

The optical freedom of this system can be integrated out to extract the effective information of atomic dynamics. Unlike the classical treatment of the optical fields in which the optical fields are recognized as plane waves with slowly varying amplitudes, the atomic motion of the system can be explicitly modified by the status of the quantized light fields, not just by the strength of the pump beams [21, 22]. To see this, we shall assume, for example, that the light fields are in number states. The Hamiltonian (5) conserves the total photon number of the system. Thus the wave function of the matter-wave condensate can be written as a linear combination of different momentum states with the corresponding photon distributions among the two light fields,

$$\left| \Psi(t) \right\rangle = \sum_n \Psi_n \left| \psi(n, N_R - n, N_L + n, t) \right\rangle,$$

where $n = 0, \pm 1, \pm 2$, refers to the diffraction order. N_R and N_L are the initial photon number distribution among the two light modes. $\left| \psi(n, N_R - n, N_L + n, t) \right\rangle$ is the product of the wave function of the atomic center-of-mass motion and that of the optical fields, $|p_n\rangle \otimes |\psi_f\rangle$. In this stage, $|\psi_f\rangle$ is choosen to be the number state noted as $|n_1, n_2\rangle$ corresponding to that there are n_1 photons in the right-going light mode and n_2 photons in the left-going mode. Since the condensate is initially prepared in the stationary state, each atom needs to absorb n photons from the right-going light mode in order to hop to the nth momentum mode. Therefore, the photon number distribution of the two light modes is directly related to the atomic diffraction order. The equation of atomic motion now reads,

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \left| \Psi(t) \right\rangle &= \hat{\mathscr{V}}(t) \left| \Psi(t) \right\rangle \\ &= \hbar G \sum_{n=-\infty}^{\infty} \{ a_{k_L}^{\dagger} a_{k_R} \left| p_n \right\rangle \left\langle p_{n-1} \right| e^{-i\delta_-(t)} \\ &\quad + a_{k_R}^{\dagger} a_{k_L} \left| p_n \right\rangle \left\langle p_{n+1} \right| e^{-i\delta_+(t)} \} \left| \Psi(t) \right\rangle. \end{aligned}$$

Keeping the photon degree of freedom, the above equation can be further simplified to,

$$\begin{aligned} \dot{\Psi}_n(t) & \left| n_R - n, n_L + n \right\rangle \\ = \quad & a_{k_L}^{\dagger} a_{k_R} \Psi_{n-1} e^{-i\delta_- t} \left| n_R - n + 1, n_L + n - 1 \right\rangle \\ & + a_{k_R}^{\dagger} a_{k_L} \Psi_{n+1} e^{-i\delta_+ t} \left| n_R - n - 1, n_L + n + 1 \right\rangle, \end{aligned}$$

Until now we have kept quantum character of the optical fields. Integrating the photon degrees of freedom by explicitly executing the operations of the photon creation and annihilation operators on the corresponding number states of the light fields, we arrive at the following equation of the atomic motion,

$$\dot{\Psi}_n(t) = W_R^n \cdot \Psi_{n-1} e^{-i\delta_- t} + W_L^n \cdot \Psi_{n+1} e^{-i\delta_+ t}, \quad (6)$$

where $W_{L,R}^n$ are the forward and backward transition weights defined as, $W_R^n = -iG\sqrt{(n_R - n + 1)(n_L + n)}$ and $W_L^n = -iG\sqrt{(n_R - n)(n_L + n + 1)}$. This equation has included the quantum effect of optical fields upon the atomic motion. The complicated expressions for the transition weights are resulted from the property of number state of the optical fields that when it is operated by the corresponding creation and annihilation operators, we have $a^{\dagger}|n\rangle = \sqrt{n+1}|n+1\rangle$ and $a|n\rangle = \sqrt{n}|n-1\rangle$. It can be directly observed from Eq. (6) that, the transition weights depend not only on the diffraction order but also on the photon distribution among the two light modes. Thus, unlike the classical case of plane wave approximation for the light fields, there is no definite equality between W_R^n and W_L^n; $|W_R^n| > |W_L^n|$ and $|W_R^n| < |W_L^n|$ are both possible, depending on whether $N_R < N_L$ or $N_L > N_R$. Table (I) lists out the transition weights of the first few order atomic scattering processes for $N_R = 80$ and $N_L = 10$. It shows that for the first few order scattering processes, $|W_R^n| < |W_L^n|$ are always satisfied.

Scattering Process	$\|W_R^n\|$	$\|W_L^n\|$	$\|W_R^n \cdot W_L^n\|$
$\|p_0\rangle \to \|p_1\rangle, \|p_{-1}\rangle$	28.46	29.66	844.12
$\|p_1\rangle \to \|p_0\rangle, \|p_2\rangle$	29.66	30.78	912.93
$\|p_{-1}\rangle \to \|p_0\rangle, \|p_{-2}\rangle$	27.16	28.46	772.97

TABLE I: The transition weights in dependence on the diffraction order in unit of atom-field coupling constant G, for the initial photon number distribution, $N_R = 80$, $N_L = 10$.

III. LIGHT FIELDS IN NUMBER STATES

A. Raman-Nath regime

To solve Eq. (6) in the full time-domain, one has to appeal to numerical calculations. Fortunately, in the Raman-Nath regime where the atoms interact with the light fields for rather short time, the low order diffractions dominant the whole atomic scattering process, and

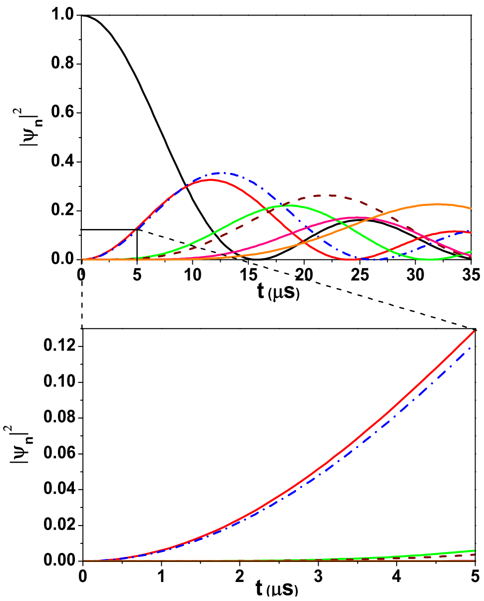

FIG. 2: (Color online). Atomic probability distribution of the first few discrete momentum modes in the limit of short light-atom interacting time, for an initial photon number distribution, $N_R = 80, N_L = 10$. Black solid curve refers to $n = 0$ mode; red solid curve , $n = 1$; blue dash-dotted curve, $n = -1$; green-solid curve, $n = 2$; wine dashed curve $n = -2$; pink solid curve, $n = 3$; orange solid curve, $n = -3$. The light-atom coupling constant is chosen $G = 0.7\nu_r$, with ν_r the atomic one-photon recoil frequency.

Eq. (6) can be solved analytically. In this limit $t \to 0$, the probability equation of the atomic motion takes the following form,

$$\dot{\Psi}_n(t) = W_R^n \cdot \Psi_{n-1} + W_L^n \cdot \Psi_{n+1}. \tag{7}$$

For macroscopic occupations of photons in the two light modes, the diffraction order n is small compared to the initial photon number N_R and N_L, i.e., $N_{R,L} \gg n$, we have $W_R^n \sim W_L^n \sim \sqrt{N_R N_L}$ which can be observed in Table (I). Thus Eq. (7) has the following solution,

$$\Psi_n(t) = i^n e^{i\theta} \left(\frac{W_R^n}{W_L^n}\right)^{\frac{n}{2}} J_n(\xi t) \tag{8}$$

where $\xi = 2\sqrt{(W_R^n W_L^n)}$, $J_n(\xi t)$ is the nth order Bessel function; θ is an arbitrary phase angle. The population of atoms occupying the nth momentum state $|p_n\rangle$ is $\rho_n(t) = |\Psi_n(t)|^2 = (\frac{W_R^n}{W_L^n})^n [J_n(\xi t)]^2$. For the inequality of W_R^n and W_L^n as show in Table (I), the atomic diffraction will display exotic behaviors. Fig. 2 shows the probability amplitudes of the atoms in different momentum modes evolving with the light-atom interacting time. It can be seen that, besides that the atomic populations in different momentum modes oscillate with time, the curves of the momentum oscillations of the $\pm n$th modes do not overlap. Unlike the situations illustrated in [21, 22], the probability amplitudes of these $\pm n$th momentum modes do not reach the minimum or maximum values exactly at the same time. For example in Fig. 2, there is a shift between the first minima of the probability amplitudes for the ± 1st order atomic momentum modes. Even though to study the atomic motion for long-period light-atom interaction from the data derived in this regime may not seem adequate enough, it implies that the atomic diffractions would be asymmetric about the zeroth order, and such asymmetries are induced by the number-state-nature of the optical fields. More detailed discussion and the accuracy of the analytical solution in this short time regime will be demonstrated in the later part of this paper.

B. Bragg regime

In the Bragg diffraction regime, where the light and atoms interact with each other for a longer time, the exponential factors in Eq. (6) greatly modify the atomic motion, compared to the situations of the short time light-atom interactions (i.e., Raman-Nath regime). However, in this regime, the equation of atomic motion cannot be explicitly solved analytically. To obtain the information of the atomic motion in the full-time domain, numerical calculation is carried out with a cutoff at the the $n = \pm 10$th momentum modes in order to maintain the accuracy. In fact, the validity of such a cutoff has been implied in the case of short-time light-atom interactions; for the very high diffraction orders, the atomic occupations in the relevant modes are always vanishingly small.

Because of the dependence of the transition weights upon the diffraction order and the photon number distribution, numerical simulation displays interesting behaviors of the atomic motion in the quantized light fields. It seems that the atoms can see the imbalanced intensities of optical fields, and thus take actions accordingly. Fig. 3 shows the probability amplitudes of atoms in the first few order momentum modes for different photon number distributions. It is found that, the quasi-period of momentum oscillations is changing with respect to the diffraction order. Fig. 3(a) shows the probability amplitudes of the scattered atoms at the photon number distribution $N_R = 80, N_L = 10$. Even for atomic diffractions of the same order, see the $n = \pm 1$ curves, the atomic populations oscillate at different frequencies. At $t \approx 28\mu s$, the curves for the $n = \pm 1$ modes get to their lowest points, but if one goes to the details in this interval, it is found that there is a shift between the minima of two curves. That is to say the populations of atoms in these two modes are in fact oscillating with different periods. Additionally, it is found that the atoms would favor the forward and backward diffractions alternatively; the $n = 1$ order is sometimes stronger and sometimes weaker than the $n = -1$ order, implying that asymmetry appears in the matter-wave diffractions. This peculiar

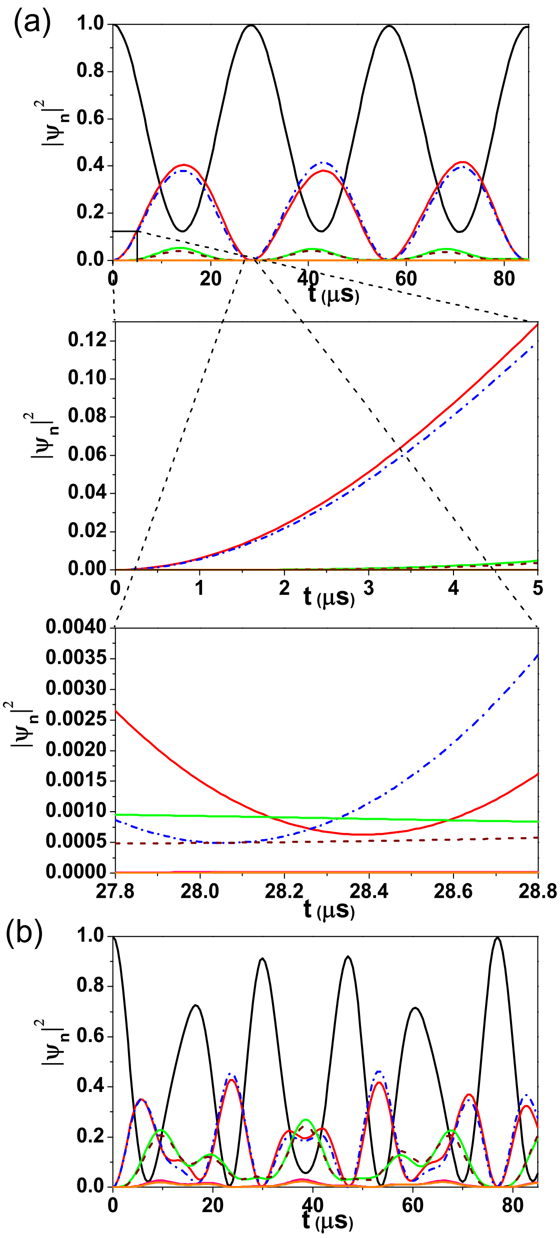

FIG. 3: (Color online). Atomic probability distribution on the first few discrete momentum modes in the regime of long light-atom interacting time, for the initial photon number distribution, (a) $N_R = 80, N_L = 10$; (b) $N_R = 80, N_L = 10$. [Line styles are the same with that in Fig 2(a)]

phenomenon is resulted from the quantum nature of the optical fields. The asymmetric atomic diffractions can be manipulated by adjusting the imbalance of the photon number distributions in the two light modes. Fig. 3(b) shows the atomic distribution of the diffracted atoms for $N_R = 200, N_L = 20$. Similar diffraction behavior is observed as that shown in Fig. 3(a). However, the atomic distributions in this case are oscillating at higher frequencies. Thus the imbalanced photon number distribution

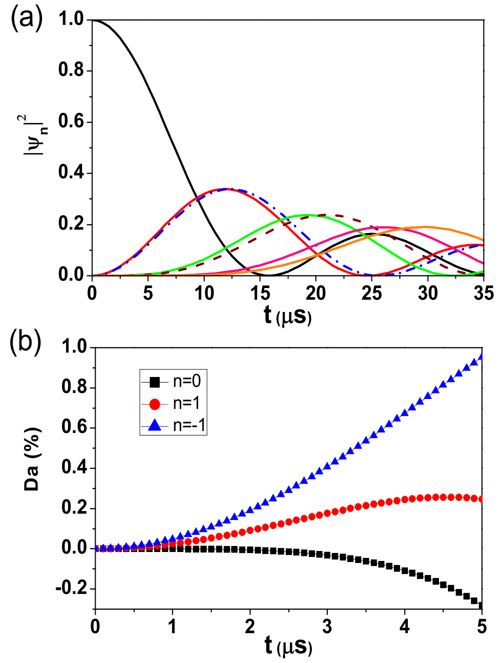

FIG. 4: (Color online). (a) Atomic probability distribution of the first few discrete momentum modes from numerical calculation in the Raman-Nath regime, for the photon number distribution $N_R = 80, N_L = 10$. [Line styles are the same with that in Fig 2(a)]. (b) The error of the analytical solution obtained in the short time limit of the light-atom interaction, compared to that obtained in the Bragg regime.

of the light fields modifies the dynamics of the atomic diffraction, and the order-dependence of the transition weights induced by the number-state-nature of the light fields makes the atomic momentum oscillation very irregular.

The above result is quite different from that obtained from the traditional analysis, where mean-field approximations are applied to both the atomic field and optical fields. The photon creation and annihilation operators are directly replaced with their mean-field values. Even though this is efficient for extracting qualitative information about the atomic dynamics, it ignores the details of influences from the quantum nature of the light fields upon the atomic motion.

To check the accuracy of solution derived in the short time limit of the light-atom interactions in the last section, numerical simulation for Eq. (7) is carried out for the photon number distribution $N_R = 80, N_L = 10$, and the result is shown in Fig. 4(a). It is found that the data obtained here is consistent with that obtained in the Bragg regime. There are displacements between the minima (or maxima) of the conjugate $\pm n$ curves, and the probability amplitude of the $n = 1$ momentum mode can be either larger or smaller than that of the $n = -1$ mode. This is similar to the result displayed in Fig. 3(a).

However, we have to emphasize that, using Eq. (7) to describe atomic motion is only valid in the Raman-Nath regime with short time light-atom interactions; thus data in Fig. 4(a) for large t is not trustful. Fig. 4(b) shows the error of the data in this regime compared to that from Fig. 3(a). The accuracy function D_a is defined as $D_a^n(t) = \frac{|\Psi_n^R|^2 - |\Psi_n^B|^2}{|\Psi_n^R|^2 + |\Psi_n^B|^2}$, where Ψ_n^R is the probability amplitude of the atomic motion in the Raman-Nath regime obtained in section II. A, and Ψ_n^B is that obtained in the Bragg regime. It is found that, for short time, $t < 5\mu s$, the error of the analytical solution Eq. (8) is less than 1%. So it is a good approximation of the atomic motion in this regime.

IV. LIGHT FIELDS IN COHERENT STATES

Besides that the properties of atomic diffractions are sensitively dependent on the intensities of the pump fields, the status of the light fields can also impose implicit corrections to the atomic diffractions. To make comparison to the case that the lights are in number states, we turn to the case that the two optical fields are in coherent sates $|\alpha_R\rangle$ and $|\alpha_L\rangle$, respectively. α_R and α_L are the eigenvalues of the annihilation operators of the two light modes on the relevant coherent states. The equation of the atomic motion reduces to

$$\dot{\Psi}_n(t) = W_{co} \cdot \Psi_{n-1} e^{-i\delta_- t} + W_{co}^* \cdot \Psi_{n+1} e^{-i\delta_+ t}, \quad (9)$$

where W_{co} and W_{co}^* are conjugate with each other, $W_{co} = \alpha_R \alpha_L^*$. Generally, α_R and α_L are complex numbers. If α_R and α_L are chosen to be real, then $W_{co} = W_{co}^* = \alpha_R \alpha_L$ and the equation is of the same form as that obtained in [21, 22]. By numerical simulation, the information of the atomic motion is obtained in the full-time domain. Fig. 5 shows the probability amplitudes of the first few momentum modes at $W_{co} = 60+40i$. It is found that the curves of the $\pm n$th orders coincide with each other; thus the forward and backward atomic transitions are equally favored, and no asymmetry appears in the atomic diffraction. This can be seen from the fact that the absolute values of transition weights for the forward and backward transitions are the same, and their phase difference can be absorbed in the definition of the phase factor $\delta_\pm$ as constant, thus the equation of the atomic motion still takes the symmetric form as that in [21, 22].

In this case, the transition weights are independent upon the diffraction order. The data obtained here resembles that with classical treatments of the optical fields, where the electromagnetic field are recognized as plane waves. Comparing the results with that in the case where the optical fields are in number states, even though the equation of the atomic motion is also derived in a full quantum treatment of the light fields, only symmetric atomic diffraction can be obtained here; this is determined by the specific status of quantized light fields.

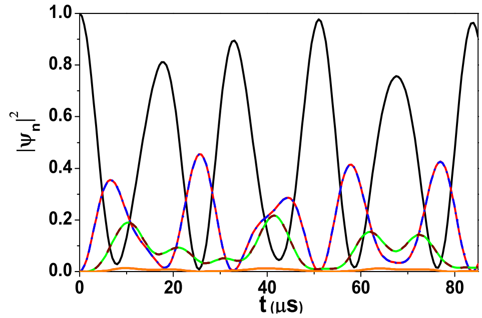

FIG. 5: (Color online). Atomic probability distribution of the first few discrete momentum modes in the condition that the light fields are in coherent sates, $|\alpha_R\rangle$ and $|\alpha_L\rangle$. The parameters are chosen such that $W_{co} = \alpha_R \alpha_L^* = 60 + 40i$. [Line styles are the same with that in Fig 2(a)].

V. SUMMARY

In this paper, we investigate the atomic diffractions in quantized light fields. The quantum nature of the light fields induces exotic phenomena in the atomic scattering processes. For light fields in number states, the atomic transitions among different discrete momentum modes depend on both the transition order and the initial photon number distribution. The quasi-periods of the atomic momentum oscillations are implicitly modified, and the atomic diffraction is no longer symmetric. However, for light fields in coherent states, the atomic diffractions are still symmetric, this verifies the coherent-state-approximation of the light fields which is frequently utilized in the cavity quantum electrodynamics. Situations of other kind of states of quantized light fields, such as arbitrary linear combination of number states, can also be investigated using the methods introduced in this paper. The results obtained in this paper can be extended to other light-atom interacting systems for enabling new physics in atom optics.

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